***Differentiation***

**Introduction:** The derivative is a mathematical operator, which measures the rate of change of a quantity relative to another quantity. The process of finding a derivative is called differentiation. There are many phenomena related changing quantities such as speed of a particle, inflation of currency, intensity of an earthquake and voltage of an electrical signal etc. in the world. In this chapter we will discuss about various techniques of derivative.

***Differentiability of a function***: The derivative of  with respect to *x* (for any particular value of x) is denoted by or and defined as,





Provided this limit exists.

***Existence of Derivative:***  A function is said to have a derivative at if the left hand derivative and right hand derivative at this point i.e,



and



are both exist and equal.

**Problem-01:** From the definition find the differential coefficient of .

**Solution:** we have 



By the definition of differentiation we have















.

**Problem-02:** From the definition find the differential coefficient of .

**Solution:** we have 



By the definition of differentiation we have

















.

**Derivatives of elementary functions:**

1. *where c is a constant.* **2.**
2.  **4.**
3.  **6.**
4.  **8.**
5.  **10.**
6.  **12.**
7.  **14.**
8.  **16.**
9.  **18.**
10.  **20.**
11.  **22.**

*where u and v are functions of x.*

* ***Find the differential coefficient*** *(**)****of the following functions with respect to x.***



**Homework:-**Find  of the following functions:

1. Ans: 
2. Ans: 
3. Ans: 
4. Ans: 
5. Ans: 
6. Ans: 
7. Ans: 
8. Ans: 
9. Ans: 
10. Ans: 

**Logarithmic differentiation:** If we have functions that are composed of products, quotients and powers, to differentiate such functions it would be convenient first to take logarithm of the function and then differentiate. Such a technique is called the logarithmic differentiation.



**Homework:-**Find  of the following functions:

1. Ans: 
2. Ans: 
3. Ans: 
4. Ans: 
5. Ans: 
6. Ans: 

**Parametric Equation:** If in the equation of a curve , *x* and *y* are expressed in terms of a third variable known as parameter i.e, then the equations are called a parametric equation.



**Homework:-**

1. Ans: 
2. Ans: 
3. Ans: 
4. Ans: 

**Successive derivative:** If be a function of *x* then the first order derivative of *y* with respect to *x*is denoted by 

Again the derivative of first ordered derivative of *y*with respect to *x*is called second order derivative and is denoted by 

Similarly, the nth derivative of *y* with respect to *x* is denoted by 

* Find the nth derivative of the following functions:



**Function of Several variables:** A function that contains more than one independent variables is called several variables function. For example  is a function of three variables x, y and z.

**Partial Differentiation:** The differentiation of a function , with respect to *x* **,** treating *y* as constant, is called the partial derivative of *u* with respect to *x*, and it is denoted as,



**Analytically**, 

when this limit exists.

Similarly, the differentiation of a function , with respect to *y* **,** treating *x* as constant, is called the partial derivative of *u* with respect to *y*, and it is denoted as, 

**Analytically**, 

provided this limit exists.

**Successive Partial Derivatives:** Consider a function , which has the partial derivatives with respect to the independent variables *x* and *y* respectively. Also each of them may possess partial derivatives with respect to these two independent variables, and these are called the second order partial derivatives of *u*, and these are denoted as,

.

Similarly, the third order partial derivatives of *u* are denoted as,

.

and so on for higher order derivatives.

**Symmetric Function:** A function  is called a symmetric function if it satisfies the condition .

**Example:** is a symmetric function.

**Problem-01:**.



Differentiating (1) partially with respect to *x* we get,







Now differentiating (2) partially with respect to *x* we get,





**(Ans.)**

Again Differentiating (1) partially with respect to *y* we get,







Now differentiating (3) partially with respect to *y* we get,





**(Ans.)**

Again Differentiating (3) partially with respect to *x* we get,





**(Ans.)**

Again Differentiating (2) partially with respect to *y* we get,





**(Ans.)**

**Problem-02:**



Differentiating (1) partially with respect to *x* we get,





Now differentiating (2) partially with respect to *x* we get,



**(Ans.)**

Again Differentiating (1) partially with respect to *y* we get,







Now differentiating (3) partially with respect to *y* we get,





**(Ans.)**

**Problem-03:**Also show that 



Differentiating (1) partially with respect to *x* we get,









Now differentiating (2) partially with respect to *x* we get,





**(Ans.)**

Again Differentiating (1) partially with respect to *y* we get,









Now differentiating (4) partially with respect to *y* we get,







**(Ans.)**

Finally, adding (3) and (5) we get,





**(Showed).**

**Problem-04:**.



Differentiating (1) partially with respect to *x* we get,









Now differentiating (2) partially with respect to *x* we get,





**(Ans.)**

Again Differentiating (1) partially with respect to *y* we get,









Now differentiating (3) partially with respect to *y* we get,





**(Ans.)**

Again Differentiating (2) partially with respect to *y* we get,







**(Ans.)**

**Problem-05:**



Differentiating (1) partially with respect to *x* we get,









Since the given function (1) is a symmetric function, so similarly differentiating (1) with respect to *y* and *z* we get,



and 

Finally adding (2), (3) and (4) we get,





**(Showed.)**

**Problem-06:**



Differentiating (1) partially with respect to *x* we get,











Since the given function (1) is a symmetric function, so similarly differentiating (1) with respect to *y* and *z* we get,



and

Finally adding (2), (3) and (4) we get,







**(Showed.)**

**Problem-07:**



Differentiating (1) partially with respect to *x* we get,









Again Differentiating (2) partially with respect to *x* we get,













Since the given function (1) is a symmetric function, so similarly differentiating (1) with respect to *y* and *z* we get,



and 

Finally adding (3), (4) and (5) we get,









**(Showed.)**

**Exercise:**

**Problem-01:** .

**Problem-02:** .

**Problem-03:** .

**Problem-04:**

**Problem-05:**

**Problem-06:**

**Problem-07:**

**Problem-08:**

|  |
| --- |
|  |

***Integration***

**Introduction:** Integration is a mathematical technique which is used to find something whose rate of change is known. In 17th century Newton and Leibnitz discovered the idea of integration. It has a wide range application in engineering, medicine, architecture, economics, etc. The objectives of this chapter are to discuss integration and provide standard integration techniques.

**Learning Outcomes**: By the end of this course, students will be able to........

(a). find displacement from velocity and velocity from acceleration.

(b). calculate areas under curves, volumes of solids, arc lengths.

(c). evaluate center of mass, moment of inertia.

(d). determine work done by a force, electric charge etc.

***Integration***: The process of finding an anti-derivative or integral of a function is called integration. It is the inverse process of differentiation. If  be a function of  related with another function in such a way that



then



which is called an indefinite integral of  with respect to .

where,and  are called integrand, integral and constant of integration respectively.

And



which is called the definite integral of  from  to , and ‘’ is called the lower limit and ‘’ the upper limit of the definite integral.

***Fundamental Properties***:

1. .
2. 

where c is a constant.

***Integration Formulas***:

1. . 2. .
2. . 4. .

5. . 6. .

1. . 8. .
2. . 10. .
3. . 12. .
4. . 14. .
5. . 16. .
6. . 18. .
7. . 20. .
8. . 22. .
9. . 24. .
10.  26. 

27.. 28. .

1. .
2. .
3. .
4. .
5. . 34. .

35.

36. 

***Illustrative Examples***:

**Problem-01**: **Exercise-01:** .

 Ans: 







where is an integrating constant.

**Problem-02**: **Exercise-02**: 

 Ans: 





where is an integrating constant.

**Problem-03:** **Exercise-03**: 

 Ans: .







where  is an integrating constant.

**Problem-04:** **Exercise-04**: 

 Ans: .















where  is an integrating constant.

**Problem-05**:  **Exercise-05**: .

 Ans:.









whereis an integrating constant.

**Problem-06**:  **Exercise-06**:.

 Ans:.









where is an integrating constant.

**Problem-07**:  **Exercise-07**:.

 Ans:.













where is an integrating constant.

**Problem-08**:  **Exercise-08**: .

 Ans:.

















where is an integrating constant.

**Problem-09**:  **Exercise-09**:

 Ans: 











where is an integrating constant.

**Problem-10**:  **Exercise-10**: 

 Ans: .

















where is an integrating constant.

**Problem-11**: 













where is an integrating constant.

***Method of substitution***

Sometimes, the integration of given integral  is relatively difficult. In this case, we can replace  by  and  by for integrating easily. This process is known as method of substitution.

**Problem-01**:  **Exercise-01**: 

 Ans: .

put



Now 







where is an integrating constant.

**Problem-02:** **Exercise-02:**

 Ans: .

put

Now 





where is an integrating constant.

**Problem-03:** **Exercise-03:**

 Ans: .

put

Now 







where is an integrating constant.

***Integration by Parts***

The formula for the integration of a product of two functions is referred to as integration by parts. *i.e,*

.

While applying the above rule for integration by parts to the product of two functions, care should be taken to choose properly the first function, i.e., the function not to be integrated.

**Problem-01:** **Exercise-01:**

 Ans: 







where is an integration constant.

**Problem-02:** **Exercise-02:**

 **Ans:** 









where is an integration constant

**Problem-03:  Exercise-03: **

 Ans: 





















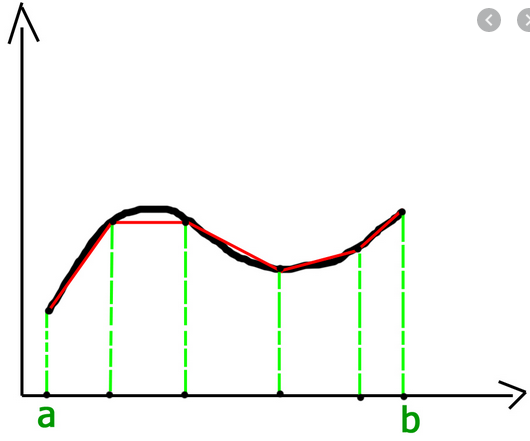
where c is an integrating constant.

**Fundamental Theorem of Integral Calculus:** Ifbe a bounded and continuous function defined in the interval where, *b*>*a* and there exists a function  such that , then



This is called the fundamental theorem of integral calculus.

**Integration as the limit of a sum:** Let, be a continuous, bounded and single-valued function defined in the interval where *a*, *b* are finite quantities and.



If the interval be divided into *n* equal sub-intervals, each of length *,* by the points  so that , then the area enclosed by is defined as

S =





 where  if then .

Which is also defined as the definite integral of with respect to *x* between the limits *a* and *b*, and is denoted by the symbol,



where, *a* is called the lower limit and *b* is called the upper limit.

Therefore,  where .

***NOTE:***

1. 
2. 
3. 

**Problem-01:** Evaluate

**Solution:** Given that,











**Problem-02:** Evaluate

**Solution:** Given that,















**Problem-03:** Evaluate

**Solution:** Given that,















**Problem-04:** Evaluate

**Solution:** Given that,





















**Problem-05:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 











.

**Problem-06:** Evaluate  from the definition of the integral as the limit of a sum.

**Solution:** We have 

Here 



Since 

where 











.

**Assignment:**

**Problem-01:** Evaluate

**Problem-02:** Evaluate

**Problem-03:** Evaluate  from the definition of the integral as the limit of a sum.

**Problem-04:** Evaluate  from the definition of the integral as the limit of a sum.

**Problem-05:** Evaluate  from the definition of the integral as the limit of a sum.

**Problem-06:** Evaluate  from the definition of the integral as the limit of a sum.

***Some Definite integrations***

**Problem-01: Evaluate** 

**Solution: Let,** 













**Problem-02: Evaluate** 

**Solution: Let,** 











**Problem-03: Evaluate** 

**Solution: Let,** 









**Problem-04: Evaluate** 

**Solution: Let,** 













**Problem-05: Evaluate** 

**Solution: Let,** 







put, 

whenthen 

whenthen 

Now, 









**Problem-06: Evaluate** 

**Solution: Let,** 





put, 

whenthen 

whenthen 

Now, 











**Area under curves (Quadrature)**

Our concentration in this Chapter is to find the area bounded by curves with a general formula or with the help of definite integration. This process is called Quadrature.

**Area formula for Cartesian equation:**

**(1).** The area bounded by the curve , the -axis and the lines andis,





Where, is a continuous single valued function and it does not change sign for .

**(2).** The area bounded by the curve , the -axis and the lines andis,

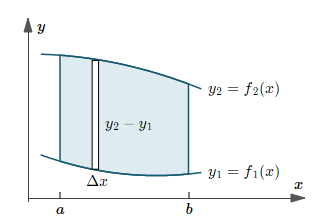




Where, is a continuous single valued function and it does not change sign for .

**(3**).The area bounded by two curves, and two vertical lines is

.



**(4).**The area bounded by the curve Symmetry about the-axis is,



**(5).**The area bounded by the curve Symmetry about the-axis is,



**Symmetry about the -axis:** If all the powers of *y* occurring in an equation are even then it is symmetry about the ****-axis. For example, ****is symmetry about the ****-axis.

**Symmetry about the -axis:** If all the powers of *x* occurring in an equation are even then it is symmetry about the ****-axis. For example, ****is symmetry about the ****-axis.

**Mathematical Problems**

**Problem 01:** Find the area bounded by the curve , the and the straight lines and .

**Solution:** We have, and.

The graph of the given curve is,

Y

X



O





The area of the region is,

****

****

****

****

** Sq. Units.**

**Problem 02:** Find the area bounded by the curve , the and the straight lines and .

**Solution:** We have, and.

The graph of the given curve is,

Y



X

O





The area of the region is,

****

****

****

** Sq. Units.**

**Problem 03:** Find the area bounded by the curve , the and the straight lines and .

**Solution:** We have, and.

The graph of the given curve is,

Y



X

O





The area of the region is,

****

****

** Sq. Units.**

**H.W:**

**1.** Find the area bounded by the curve , the and the straight lines and .

**2.** Find the area bounded by the curve , the and the straight lines  and .

**3.** Find the area bounded by the curve , the and the straight lines  and .

**Problem 04:** Find the area of the region bounded by the curve ; from and .

**Solution:** We have,and.

Since, only even power of *y* occurs in the given curve so the curve is symmetric about the *x*-axis.

The graph of the given curve is,

Y



X

O



Also, the given curve can be written as,





The area of the region is,

****

****

****

****

****

****

****

** Sq. Units.**

**Problem 05:** Find the area of the region bounded by the curve  and *y*-axis.

**Solution:** We have,

Since, only even power of *y* occurs in the given curve so the curve is symmetric about the *x*-axis.

The graph of the given curve is,

Y



X

O







Putting in (1) then we have  , so the vertex is at .

Also putting in (1) then we have. So the curve crosses the *y*-axis at andThe given curve can be written as,





The area of the region is,

****

****

****

****

****

****

****

****

** Sq. Units.**

**H.W:**

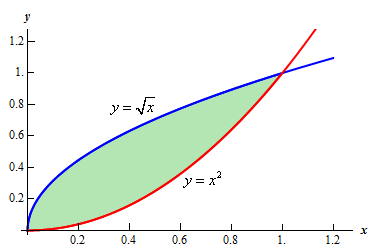
1. Find the area of the region bounded by the curve ; from and .

2. Find the area of the region bounded by the curve ; from and .

**Problem 06:** Find the area of the region enclosed by and.

**Solution:** The equation of the given curves are  and.

The graph of the given curves is as follows:



We have

and

Now,



 [Squaring both sides]







Therefore, and 











For real we get respectively 

Therefore, the given curves intersect each other in two point at and.

In the question, **.**

So, the area of the region is,

****

****

****

****

****

****

****

****

** Sq. Units. ( As desired)**

**Integration by Partial Fraction**

**Rational Fraction:** If  are two polynomials in  and  then the quotient  is called a rational fraction.

**Example**:  is a rational fraction.

**Proper Fraction**: A fraction in which the degree of the numerator is less than the degree of denominator is called a proper fraction.

**Example**:  is a proper fraction.

**Improper Fraction**: A fraction in which the degree of the numerator is greater or equal to the degree of denominator is called an improper fraction.

**Example**:  are improper fractions.

**Partial Fraction**: A given fraction may be written as a sum of other fractions (called partial fractions) whose denominator is less than the denominator of the given fraction.

**Fundamental theorem**: Any fraction may be written as the sum of partial fractions according the following rules:

**Case-1**: When the fraction is **Proper fraction**:

1. When all factors are linear and different

i.e.



where the coefficients of the blank spaces cannot be zero.

**NOTE**: Using the **Cover up method** we can find the values of the blank spaces of (1).

**Cover up method**: This method is applicable only for linear factors.

If  then

For A: Cover term in the denominator of the left-hand side and substitute  in the remaining expression.

For B: Cover term in the denominator of the left-hand side and substitute  in the remaining expression.

1. When all factors are linear and some are repeated

i.e.



**NOTE**: Find the coefficients of the blank spaces by using **Cover up method** and then to find  substitute any value for except .

1. When all factors are quadratic and different

i.e.



**NOTE**: To find the values of, , &multiplying both sides of (3) by and then substitute the appropriate values for .

1. When all factors are quadratic and some are repeated

i.e.,



**NOTE**: To find the values of , , , , &multiplying both sides of (4) by and then substitute the appropriate value for .

**Case-2**: When the fraction is **improper fraction**: To split an improper fraction into a partial fraction, we will have to divide the numerator by denominator.

**Example**: If  then



Rewriting the given improper fraction we get



Now using the Cover up method anyone can solve the fraction.

**Problem-01:** Evaluate.

**Solution**: Let 











where c is an integrating constant.

**Problem-02:** Evaluate.

**Solution**: Let 

Here 

Putting  in (1) we get,













From (1) we get,



Now 



**Problem-03:** Evaluate.

**Solution**: Let 

Here 

Multiplying both sides by , we get









Equating the coefficients of , and constant terms we get,





From (1) we get,





Now 





where c is an integrating constant.

**Problem-04:** Evaluate.

**Solution**: Let 

Here 

Multiplying both sides of (1) by we get,







Equating the coefficients of like term we get,





Since  so 





 and

Again so 





and

From (1) we get,





Now 







where c is an integrating constant.

**Problem-05:** Evaluate.

**Solution**: Let 

Here 











Now 



where c is an integrating constant.

**Exercise:**

1. Evaluate .
2. Evaluate .
3. Evaluate .
4. Evaluate.
5. Evaluate .
6. Evaluate .
7. Evaluate .
8. Evaluate .